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EFFECT OF ATMOSPHERIC TURBULENCE ON AN AIRPLANE WITH FLEXIBLE WINGS AT DIFFERENT SPEEDS OF FLIGHT

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EFFECT OF ATMOSPHERIC TURBULENCE ON AN AIRPLANE WITH FLEXIBLE WINGS AT DIFFERENT SPEEDS OF FLIGHT

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The varying part of the lifting force acting on an elastic vibrating wing of an airplane in flight can be broken down into two components: 1) the component which represents the varying part of the lifting force of an airplane wing under conditions of flight in ideally quiet air, and 2) the random lifting force brought about by atmospheric turbulence. If one limits oneself to the theory of the vibrations of an elastic airplane in a turbulent flow, the problem of the vibrations of an airplane in a turbulent flow of air can be described with the aid of the following block diagram (Fig. 1). Here the vector ε characterizes the elastic vibrations of the airplane, the vector q determines the aerodynamic loads produced as a result of these vibrations, and the vector p represents the aerodynamic loads arising from the presence of the turbulence of the air. Meanwhile all these values ε, q, and p are interrelated with the aid of linear, generally integrodifferential equations which depend on the horizontal speed of flight.

The properties of the airplane-air system change with the change in the speed of flight. Even with complete absence of turbulence at a determined value for the critical speed in the system, the block diagram of which is shown in Fig. 1, there may appear movements connected with elastic vibrations, these movements having increasing amplitude (for example, wing flutter). Such instability is explained by the fact that the feedback shown in the block diagram becomes positive at a determined value for the coefficients which determine the feedback.

The goal in this work is the investigation of forced vibration of an elastic airplane wing under the action of turbulence as depends on the speed

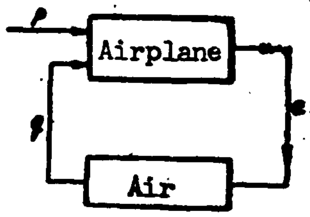

Fig. 1

of flight. The investigation deals with airplanes with straight wings which have considerable length in subsonic flight speed. Therefore there is taken as a basis of the computation of the aerodynamic loads the theory of the movement of a wing of infinite length in a plane flow [1].

With some limitations, the outside fluctuation action on the wing may be considered to be a stationary normal random process depending on the time [2]. By this same principle, and because the whole system is assumed to be linear, one can make a description of its movement on the basis of the correlation theory, and particularly on the basis of the theory of of the passing of stationary random signals through linear systems (see, for example, [3]).

In this way the vibrations of the airplane in the vicinity of the position of equilibrium and the elastic vibrations of its wings under the action of atmospheric turbulence can be presented as some stationary, normal, random process. The investigation of this process amounts to a determination of its different statistical characteristics.

The action of the random gusts of wind on the wing of an airplane can be presented in the following fashion. In its movement with a horizontal speed of U the airplane goes through rising and descending currents of air, and the values for the velocities of these currents change arbitrarily both in one point in space relative to another and in time. The change in the value for the velocity of a gust of wind in a given point in space at the time of an airplane wing's passing through it can be disregarded. Random change in the vertical component of a turbulent velocity v in the displacement of an airplane from one point in space to another leads to fluctuations in the effective angle of attack, and these determine the occurrence of random lifting force. The influence of the horizontal component of the random velocity of

gusts of wind ordinarily are disregarded, since they are much less than the speed of the airplane. Close to the critical states the disregarding of the horizontal component must be substantiated because it determines the parametrical random action on the wing and can lead to parametrical excitation of vibrations of the wing in the turbulent flow. As to the question of the action on the wing of the horizontal component of the velocity, the authors intend to devote a separate study to this.

The most effective action is brought to bear on the airplane by the large-scale components of the turbulence. In the studies [2, 4] there is presented some information on large-scale turbulence. All the authors indicate that the atmospheric turbulence in the first approximation can be considered as isotropic. The radius of the correlation of velocities by various data fluctuates between 100 and 200 m. The figures for magnitudes of the spectral densities depend to a very great extent on the meteorological conditions, the topography of the locality and the altitude of the flight. In Fig. 2 there are presented graphs of the function of the spatial spectral density $S[(m^2/sec^2)/(rad/m)]$ for the vertical component of the velocity of the turbulence v, as depends on the spatial frequency Ω [rad/m] on the basis of data presented in [2]. At the time of thunderstorms the intensity of the turbulence may exceed the indicated values by one order.

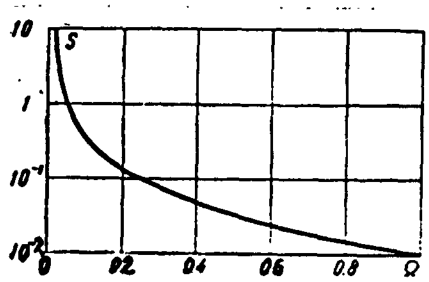

Fig. 2

A number of studies have been devoted to the question of the action of atmospheric turbulence on an aircraft using the methods of the theory of random processes. In the study [3] there is used the transmissive function of a linear system for determining the root-mean-square value of the spectral characteristics of the lifting force of a rigid airplane wing. In the study [5] it is shown that taking into account the change in the velocity

of the turbulence along the wing leads to a decrease in the effect of the action on the wing on the part of the turbulence. This change is not great. Therefore from here on it will be assumed that along the wings at each given moment of time the velocity is constant. In the studies [5, 6] there is taken into account the effect of the elasticity of the wing on the bending in determining the statistical characteristics of normal acceleration of the bending moment of the wing. There exist indications to the effect that in the U. S. A. computations were made of the bending and torsional moments acting in the root of the stabilizer of the aircraft as depends on the speed of flight [7].

1. Equations of the System In writing the equations which describe the vibrations of the airplane in its flight the following scheme was followed. The free airplane flying in the air produces elastic vibrations with bending and torsional deformation of the wing with vertical vibrations of the fuselage. In this scheme the fuselage is considered to be a rigid body with a finite mass (comparable with the mass of the wing) and possessing an infinitely great moment of inertia relative to the axis of rigidity of the wing (more precisely a great moment of inertia as compared with the same magnitude for the wing). Therefore the fuselage produces only vertical vibrations as a solid, and by virtue of the symmetry of the airplane one may consider only one wing with the half of the mass of the whole fuselage on one end. From here on we will consider only symmetrical vibrations.

As initial vibrations of the wing, as is usually done [8], there are taken the equations of the bending-torsional vibrations of a quite long beam which practically well represents the elastic vibrations of a wing with great extension

$$\begin{gathered}\frac{\partial^2}{\partial x^2}\left[EI\frac{\partial^2 Z}{\partial x^2}\right]+m\frac{\partial^2 Z}{\partial t^2}-m\sigma\frac{\partial^2\theta}{\partial t^2}=f_1(x,t)+f_2(x,t)\\ -\frac{\partial}{\partial x}\left[GI_p\frac{\partial\theta}{\partial x}\right]+m\sigma\frac{\partial^2 Z}{\partial t^2}+I_m\frac{\partial^2\theta}{\partial t^2}=M_1(x,t)+M_2(x,t)\end{gathered}\qquad(1.1)$$

where Z(x,t) is the vertical displacement of the axis of rigidity of the

wing relative to the neutral position, and $\Theta(x,t)$ is the angle of turn of the section of the wing around the axis of rigidity. The positive directions are shown in Fig. 3. All sections of the wing are considered to be rigid and nondeformable under oscillations. The linear mass of the wing is $m(x)$ and the statistical moment $m(x)\sigma(x)$ and the moment of inertia $I_m(x)$ relative to the axis of rigidity are linear. EI and GI_p are the characteristics of the rigidity of the wing to bending and torsion, respectively, and f_1 is the varying part of the linear aerodynamic lifting force and its moment M_1; f_2 and M_2 are the random aerodynamic force and moment which come into existence as a result of the turbulence.

The system of equation (1) should be satisfied under the following conditions
$$\theta(x,t)\Big|_{x=0}=0,\quad \frac{\partial\theta(x,t)}{\partial x}\Big|_{x=l}=0,\quad \frac{\partial^2 Z(x,t)}{\partial^2 x}\Big|_{x=l}=0$$
$$\frac{\partial}{\partial x}\left[EI\,\frac{\partial^2 Z(x,t)}{\partial x^2}\right]\Big|_{x=l}=0,\quad \frac{\partial Z(x,t)}{\partial x}\Big|_{x=0}=0,\quad \int_0^l mZdx+\frac{m_0}{2}Z(0,t)=0 \qquad (1.2)$$
where l is the length of the wing and M_0 is the mass of the fuselage.

Obtaining a precise solution of the system (1.1) appears not to be possible. For finding approximation solutions, as usual we will use the method of Bubnov and Galerkin. The movement of the wing we will limit by the following conditions: the wing may bend symmetrically in accordance with a definite selected form and twist according to a definite selected form. The curved form corresponds to the form of that of the fundamental of natural symmetrical bending oscillations of a cantilever wing, and the form of the torsional oscillations to that of the fundamental of the natural torsional oscillations of a wing fixed in cantilever form.

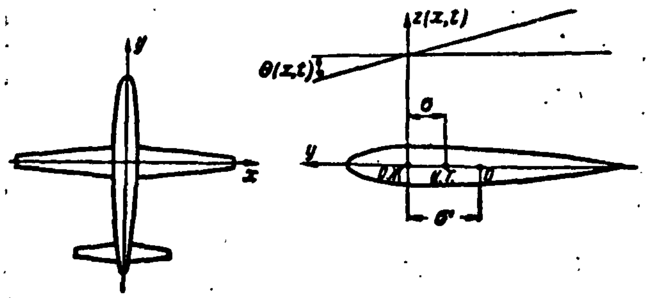

Fig. 3

With these conditions the solution can be written
$$Z(x,t)=a_0(t)+a_1(t)(Z_1 t),\quad \theta(x,t)=a_2(t)\,\theta_2(x) \qquad (1.3)$$

where a_0, a_1, and a_2 are the new variables (according to Galerkin). Meanwhile $Z_1(x)$ and $\theta_2(x)$ are the forms of the natural movements of the wing.

By substituting the solution in the form (1.3) in the equation (1.1) and using the known procedure for the functions a_0, a_1, and a_2 we will get the system

$$\begin{aligned}
&\frac{d^2a_0}{dt^2}\int_0^l m\,dx-\frac{d^2a_2}{dt^2}\int_0^l m\sigma\theta_2\,dx=\int_0^l f_1\,dx+\int_0^l f_2\,dx\\
&\frac{d^2a_1}{dt^2}\int_0^l mZ_1^2\,dx-\frac{d^2a_2}{dt^2}\int_0^l m\sigma Z_1\theta_2\,dx+\\
&\qquad+a_1\int_0^l \frac{\partial^2}{\partial x^2}\left[EI\frac{\partial^2 Z_1}{\partial x^2}\right]Z_1\,dx=\int_0^l f_1Z_1\,dx+\int_0^l f_2Z_1\,dx\\
&\frac{d^2a_2}{dt^2}\int_0^l I_m\theta_2^2\,dx-\frac{d^2a_0}{dt^2}\int_0^l m\sigma\theta_2\,dx-\frac{d^2a_1}{dt^2}\int_0^l m\sigma Z_1\theta_2\,dx-\\
&\qquad-a_2\int_0^l \frac{\partial}{\partial x}\left[GI_p\frac{\partial\theta_2}{\partial x}\right]\theta_2\,dx=\int_0^l M_1\theta_2\,dx+\int_0^l M_2\theta_2\,dx.
\end{aligned}\tag{1.4}$$

For solving the problem within the framework of the correlation theory it is sufficient to get a solution of the system (1.4), where instead of the outside random force f_2 and the moment M_2 there are given the force and the moment resulting from the action on the wing of the airplace of a disturbance produced by the presence of the harmonic wave of the vertical speed

$$v=V_0\exp\left[j\left(\omega t-\frac{2ky}{b_0}\right)\right]=V_0\exp\left[j\left(\Omega Ut-\frac{2ky}{b_0}\right)\right]\qquad\left(k=\frac{\omega b_0}{2U}\right)\tag{1.5}$$

with a frequency ω and phase speed U relative to the airplane (from here on we will consider that k is Strouhal's number).

Since the vertical speed v of the "cantilever disturbance" is given in the form (1.5), therefore in the system studied, as a result of its linearity, there are possible only harmonic movements. This means that in the case of the flight of an airplane in "harmonically disturbed" air there are formed stationary harmonic oscillations with a frequency ω. Therefore the stationary solutions of the system (1.4) should be sought in the form

$$a_0=e^{j\omega t}(A_0+jB_0),\qquad a_1=e^{j\omega t}(A_1+jB_1),\qquad a_2=e^{j\omega t}(A_1+jB_2)\tag{1.6}$$

where A_0, B_0, A_1, B_1, A_2, and B_2 are the amplitude coefficients depending on the frequency ω, V_0, U, and the parameters of the airplane.

By substituting (1.6) in (1.4) we will obtain the equation for the

amplitude coefficients in the form in which they are presented in [1].

$$
\begin{gathered}
(A_0+jB_0)\left[-\frac{2k^2}{b_0^2}\int_0^l m\,dx+\frac{2}{b_0}jkC(k)\pi\rho\right]+(A_1+ \\
+jB_1)\ jkC\ (k)\frac{2}{b_0}\pi\rho\int_0^l bZ_1dx+(A_2+jB_2)\left[k^2\frac{4}{b_0^3}\int_0^l m\sigma\theta_2dx+\right. \\
\left.+jk\frac{2}{b_0}\pi\rho\int_0^l b^2\theta_2\,dx\left(-\frac{1}{4}-\Delta_1C\ (k)\right)-\pi\rho C(k)\int_0^l b\theta_2dx\right]=\frac{\pi\rho}{U}V_0\Psi(k)\int_0^l bdx \\
(A_0+jB_0)\ jk\pi\rho C(k)\frac{2}{b_0}\int_0^l bZ_1dx+(A_1+jB_1)\left[-k^2\frac{4}{b_0^2}\int_0^l mZ_1^2dx+\right. \\
\left.+\pi\rho jkC(k)\frac{2}{b_0}\int_0^l bZ_1\,dx+\frac{1}{U^2}\int_0^l\frac{\partial^2}{\partial x^2}\left[EI\frac{\partial^2Z_1}{\partial x^2}\right]Z_1\,dx\right]+ \\
+(A_2+jB_2)\left[k^2\frac{4}{b_0^2}\int_0^l m\sigma Z_1\theta_2dx-jk\frac{2}{b_0}\pi\rho\int_0^l b^2\theta_2Z_1\,dx\left(\frac{1}{4}+\Delta_1C(k)\right)-\right. \\
\left.-\pi\rho C(k)\int_0^l bZ_1\theta_2dx\right]=\pi\rho\frac{V_0}{U}\Psi(k)\int_0^l bZ_1\,dx \\
(A_0+jB_0)\left[\frac{4}{b_0^2}k^2\int_0^l m\sigma\theta_2+\frac{2}{b_0}\pi\rho jkC(k)\int_0^l b^2\theta_2dx\right]+ \\
+(A_1+jB_1)\left[\frac{4}{b_0^2}k^2\int_0^l m\sigma Z_1\theta_2\,dx+\pi\rho jk\frac{2}{b_0}C(k)\Delta_2\int_0^l b^2Z_1\theta_2dx\right]+ \\
+(A_2+jB_2)\left[-k^2\frac{4}{b_0^3}\int_0^l I_m\theta^2{}_2dx+jk\frac{2}{b_0}\pi\rho\Delta_1\int_0^l b^3\theta_2^2\,dx\left(\frac{1}{4}-\Delta_2C(k)\right)-\right. \\
\left.-\frac{1}{U^2}\int_0^l\frac{\partial}{\partial x}\left[GI_p\frac{\partial\theta_2}{\partial x}\right]\theta_2dx-\pi\rho C(k)\int_0^l b^2\theta_2^2\ dx\right]=\pi\rho\frac{V_0}{U}\Delta_2\Psi(k)\int_0^l b^2\theta_2\,dx
\end{gathered}
\tag{1.7}
$$

Here ο is the density of the air, ο' is the averaged distance from the center of the chord to the axis of rigidity, C(k) is Theodorsen's function [1], (k) is Sears's function, and b(x) is the chord of the wing (Fig. 3b)

$$
\Delta_1=\frac{1}{2}\left(\frac{1}{2}-a\right),\qquad \Delta_2=\frac{1}{2}\left(\frac{1}{2}+a\right),\qquad a=2\sigma'/b^0 \tag{1.7a}
$$

By dividing the imaginary and real parts in (1.7) it is possible, instead of a system of three equations with complex coefficients, to write a system of 6 linear algebraic equations relative to the unknowns A_o, B_o, A_1, B_1, A_2, and B_2. In the matrix form of expression such a system will have the form

$$
\begin{Vmatrix} a_{11} & \ldots & a_{16} \\ a_{21} & \ldots & a_{26} \\ a_{31} & \ldots & a_{36} \\ a_{41} & \ldots & a_{46} \\ a_{51} & \ldots & a_{56} \\ a_{61} & \ldots & a_{66} \end{Vmatrix}
\begin{Vmatrix} A_0 \\ B_0 \\ A_1 \\ B_1 \\ A_2 \\ B_2 \end{Vmatrix}
=
\begin{Vmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \end{Vmatrix}
\tag{1.8}
$$

The coefficients a_{ik} and b_i are functions of the number k. Therefore

also the solution of the system (1.8) depends on k. It is necessary to find A_o, B_o, A_1, B_1, A_2, and B_2 as functions of k or frequency transmission characteristics of "output values" (a_o, a_1, and a_2) which determine the vibrations of the aircraft relative to the "input values" of the speed determined by (1.5).

The stationary vibrations of the airplane in case of "harmonic disturbance" to a great extent are determined by the properties of the determinant $[a_{ik}]$. In those areas of change of parameters where this determinant takes on the form of minimum values there will generally occur an increase in the amplitude coefficients, increase in the stationary vibrations of the wing. Such "resonance" is expressed most clearly on the approach to the conditions of flutter of the airplane.

Investigation of the behavior of the solutions of the system (1.8) in the general form is not possible because of their extreme bulkiness. For solving the system (1.8) and obtaining the values of the amplitude coefficients for concrete airplanes, as depends on the numbers k and U and the parameters of the airplane, there was prepared a standard program by the aid of which on a rapid-action digital electronic computing machine of the type "Strela" it was possible to obtain numerically the values of the amplitude coefficients with 28 different values for k and four values of U with $V_o = 1$.

2. Statistical Characteristics of Forced Vibrations of a Wing In designing a wing for strength practical interest is afforded by the statistical characteristics of the bending and torsional moments of the wing, namely: 1) the spectral density of these values, 2) their root-mean-square values, 3) the number of cases of exceeding (by the indicated values) some given level per unit of time.

For the spectral densities of the bending M and the torsional τ of the moments in the root of the wing, by the rules of the correlation theory, we have

$$S_M(k) = \left(EI\frac{\partial^2 Z_1}{\partial x^2}\Big|_{x=0}\right)^2 S_z(k) = \left(EI\frac{\partial^2 Z_1}{\partial x^2}\Big|_{x=0}\right)\frac{A_1^2(k)+B_1^2(k)}{U^2}S(k) \tag{2.1}$$

$$S_\tau(k) = \left(GI_p\frac{\partial\theta_2}{\partial x}\Big|_{x=0}\right)^2 S_\theta(k) = \left(GI_p\frac{\partial\theta}{\partial x}\Big|_{x=0}\right)\frac{A_2^2(k)+B_2^2(k)}{U^2}S(k)$$

where S(k) is the spectral density of the vertical component of the velocity of the turbulence.

The root-mean-square values of Σ_M and Σ_τ of these moments are equal to

$$\Sigma_M = \left[\int_0^\infty S_M(k)\,dk\right]^{1/2}, \qquad \Sigma_\tau = \left[\int_0^\infty S_\tau(k)\,dk\right]^{1/2} \tag{2.2}$$

The root-mean-square values of the amplitude of the bending and torsional vibrations of the end of the wing Σ_Z and Σ_θ are represented in a similar way

$$\Sigma_Z = \left[\int_0^\infty S_Z(k)\,dk\right]^{1/2}, \qquad \Sigma_\theta = \left[\int_0^\infty S_\theta(k)\,dk\right]^{1/2} \tag{2.3}$$

On the end of the wing $Z_1(1) = \Theta_2(1) = 1$.

Finally, the frequency of the excesses of $N_\varphi(\xi)$ of some function $\varphi(t)$ fixed by the level ξ per second is given by Rice's formula [3]

$$N_\varphi(\xi) = \frac{1}{2\pi}\exp\left(-\frac{1}{2}\frac{\xi^2}{\Sigma_\varphi^2}\right)\left[\int_0^\infty S_\varphi(k)\,k^2dk\right]^{1/2}\left[\int_0^\infty S_\varphi(k)\,dk\right]^{-1/2} \tag{2.4}$$

By a similar system one can determine the statistical characteristics of other functions which describe the movement of the airplane, for example, of the normal acceleration $\partial^2 Z(x,t)/\partial t^2$, etc.

3. <u>Investigation of the Statistical Characteristics of the Forced Vibrations of the Model of an Airplane</u> For the investigation of the statistical characteristics with the aid of an electronic computing machine a concrete model of an airplane was taken. The numerical values of all the basic parameters, and also the forms of the bending and torsional vibrations $Z_1(x)$ and $\Theta_2(x)$ for the said model were taken from [10], page 214. The model is distinguished from an airplane by the lowered rigidity with regard to torsion and bending, and therefore it has its own frequency of bending f_z and torsion

$$f_z = \omega_z/2\pi = 3.5\ \text{гц}, \qquad f_\theta = \omega_\theta/2\pi = 5.04\ \text{гц} \tag{2.4a}$$

The decreased natural frequency is connected with the fact that the intensity of the turbulence spectrum is greatly reduced with the frequency. It was important to follow the effect of the turbulence in the case where the frequency of the flutter of the model still lies in that area of frequencies of the

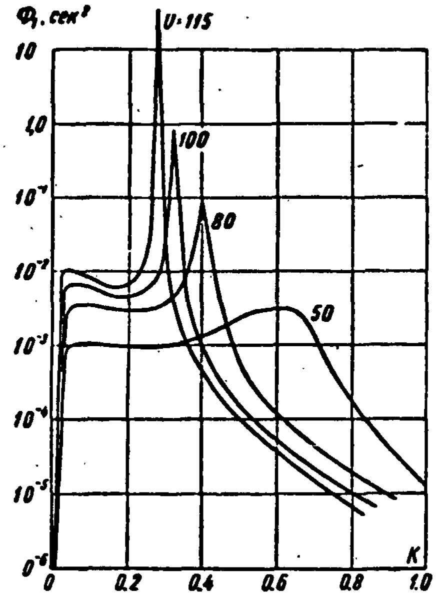

Fig. 4

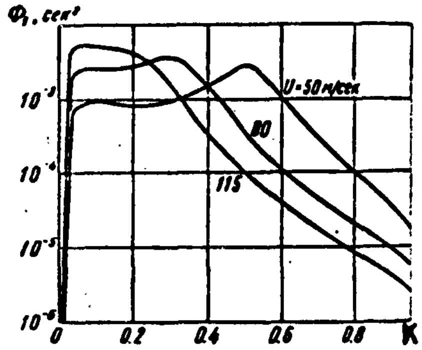

Fig. 5

turbulence spectra where the intensity is relatively great.

Let us present some numerical values of the basic coefficients of equation (1.7) of the model under investigation.

Half of mass of airplane model $M = 2063$ kg

Statistical moment of the wing $S' = 77$ kgm

Moment of inertial of the wing $I_{mo} = 135$ kgm^2

Average chord $b_o = 2.3$ m

Average distance from rigidity line to central line $a = 2\sigma/b_o = -0.46$

Average statistical moment in the end of the wing $M_o = 6{,}000$ kGm

Half of wing span $l = 6.12$ m

The magnitudes of the united masses of air and their inertia are included in the respective coefficients.

There were computed the squares of the modules of the transmission characteristics for the bending and torsion of the wing

$$\Phi_1(k) = \frac{A_1^2(k) + B_1^2(k)}{U^2}, \qquad \Phi_2(k) = \frac{A_2^2(k) + B_2^2(k)}{U^2} \tag{2.4b}$$

of the model under investigation with several values for the speed (the computation was done on the "Strela" in the Computation Center at the Moscow State University; for obtaining 108 solutions of the system of equations of the 6th order (13) five minutes were required).

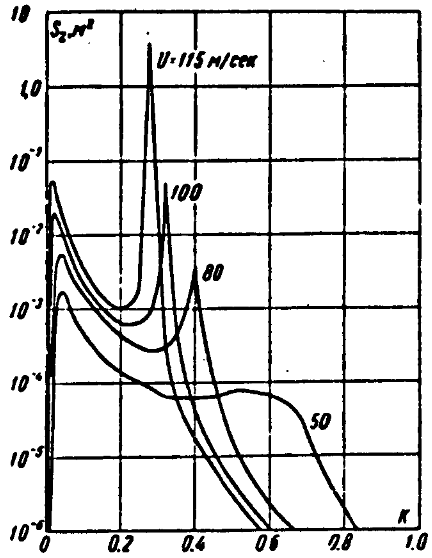

Fig. 6

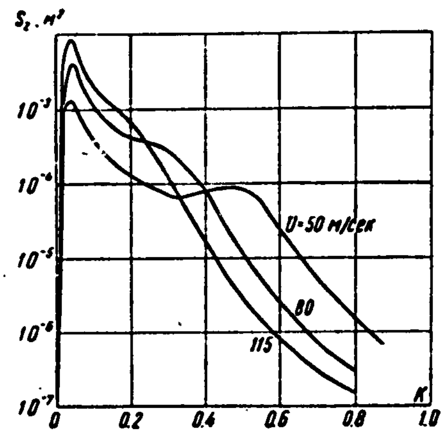

Fig. 7

In order to determine the influence of torsion on these transmission characteristics analogous computations were made for the same wing in the proposition $\theta_2(x) \equiv 0$. In Fig. 4 there are presented graphs of the function $\Phi_1(k)$ for the system with 6 degrees of freedom. In Fig. 5 there are the corresponding graphs of the system in the proposition $\theta_2 \equiv 0$ (four degrees of freedom). The critical rate of the flutter U_*, computed for the model, amounted to about m/sec.

From a comparison of the transmission functions (Figs. 4 and 5) and the curves of the spectral densities of the bending of the end of the wing for the systems with six and four degrees of freedom (Figs. 6 and 7) constructed for different values of U m/sec, it follows that taking into account the torsion of the wing leads to a sharp accentuation of the curves of the transmission functions, and, accordingly, of the curves of the spectral densities with an increase in the rate of U.

In the tables 1 and 2 there are presented confrontations of statistical characteristics obtained on the basis of the curves of spectral densities by the formulas (2.2), (2.3), and (2.4) for the case $U_* = 117$ m/sec. With the aid of the data of these tables one can follow the change in the root-mean-square of the values $\Sigma_M, \Sigma_\tau, \Sigma_z$, and Σ_θ, and of the functions $N_z(\xi)$ and $N_\theta(\xi)$

T a b l e 1

Displacement – bending (ξ is the deflection of the end of the wing in cm)

U, м/сек	Σ_Z, см	Σ_M, кГм	$N_z(\xi)$, $\xi = 0$	$N_z(\xi)$, $\xi = 4$	$N_z(\xi)$, $\xi = 15$
50	1.18	342	1.4	0.45×10^{-3}	0.28×10^{-35}
80	1.61	467	1.8	0.81×10^{-1}	0.54×10^{-10}
115	2.54	735	1.6	0.47	0.47×10^{-7}
		(1)	0	1160	4350

Key: (1) moment in the root of the wing with a given bending ξ of the end of the wing (kg-m)

with the change in the rate of U. In both the cases considered with an increase in U there occurs an increase in the computed root-mean-square values and the numbers of intersections of a given level. In the first case (displacement – bending – torsion) the root-mean-square values of Σ_Z with the increase in the rate of U from 50 to 115 m/sec are increased by a factor of 12, while in the second case (displacement – bending) Σ_Z is increased only by a factor of 2 with the same change in the speed. The number of cases of exceeding a given level with the growth in speed changes still more.

T a b l e 2

Displacement – bending – torsion (ξ is the deflection of the end of the wing in cm, ξ_1 is the angle of turn of the wing at the end)

U, м/сек	Σ_z, см	Σ_M, кГм	Σ_θ, град (1)	Σ_τ, кГм	$N_z(\xi)$, ξ, см: 0	$N_z(\xi)$, ξ, см: 4	$N_z(\xi)$, ξ, см: 15	$N_\theta(\xi_1)$, ξ, градусы (2): 0	$N_\theta(\xi_1)$, ξ, градусы (2): 0.4	$N_\theta(\xi_1)$, ξ, градусы (2): 2.0
50	1.19	345	0.07	7.7	1.67	0.6×10^{-2}	0.8×10^{34}	4.28	0.47×10^{-6}	0.77×10^{-171}
80	2.1	608	0.15	16.7	2.47	0.36	2.5×10^{-11}	4.03	1.25×10^{-1}	0.65×10^{-37}
100	4.0	1160	0.31	34.1	3.11	1.9	2.64×10^{-3}	4.31	1.9	3.23×10^{-9}
115	14.5	4210	1.41	155	4.56	4.4	2.7	4.5	4.35	2.02
							(3)	0	44	220

Key: (1) degree; (2) degrees; (3) moment in the root of a wing section with a given angle of torsion ξ_1 on the end (kgm)

Analogous computations were made for the same model of airplane with changed natural frequency of torsion ($f_z = 3.5$ cps, $f_o = 6.0$ cps, $U_* = 200$ m/sec).

Meanwhile the character of the change of the statistical characteristics proved to be about the same as in the first case.

Conclusions The described method makes it possible to obtain simply the statistical characteristics of forced vibrations of an elastic airplane wing under the action of atmospheric turbulence. With its help one can take into account vibrations with different degrees of freedom. The increase in the number of degrees of freedom per unit leads to an increase in the order of solvable system of algebraic equations by two units. Meanwhile into the standard program it is necessary to introduce insignificant changes; the time for the computation correspondingly is increased by a small factor.

Comparison of the statistical characteristics of vibrations of one and the same model of airplane shows that with sufficiently low frequencies of torsion of a wing close to the critical rates of flutter one should not disregard the torsion of the wing, but it is necessary to take it into consideration in computing the joint bending-torsional vibrations of the wing.

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Literature

1. A. K. Nekrasov. Theory of a Wing in a Nonstationary Flow, Pub. Off. of the Acad. of Sci. of the USSR, Moscow - Leningrad, 1947.

2. H. Press and J. C. Houbolt. Some Applications of Generalized Harmonic Analysis to Gust Loads on Airplanes, J. Aeronaut. Sci., No. 1, V. 22, pp. 17 - 26, 1955.

3. P'ien Hsüeh Sên, Technical Cybernetics, Foreign Literature, 1956.

4. M. Kramer. Turbulence Measurements in Flight, J. Aeronaut. Sci., No. 9, V. 20, pp. 655 - 656, 1953.

5. F. W. Diederich. The Dynamic Response of a Large Airplane to Continuous Random Atmospheric Disturbances, J. Aeronaut. Sci., No. 10, V. 23, pp. 917 - 931, 1956.

6. J. C. Houbolt and E. E. Kordes. Structural Response to Discrete and Continuous Gusts of an Airplane Having Wing Bending Flexibility and a Correlation of Calculated and Flight Results, Report 1181, NACA, 1954.

7. B. Mazelsky and R. F. O'Connel. The Integrated Use of Analog and Digital Computing Machines for Aircraft Dynamic Load Problems, J. Aeronaut. Sci., No. 8, V. 23, 1956.

8. E. P. Grossman. Course of the Vibrations of Parts of an Airplane, Oborongiz (Pub. Off. for Lit. of the Defense Ind.), 1940.

9. N. H. Kemp. On the Lift and Circulation of Airfoils in Some Unsteady Flow Problems, J. Aeronaut. Sci., No. 10, V. 4, p. 713, 1952.

10. R. Scanlan and R. Rosenbaum. Introduction to the Study of Aircraft Vibration and Flutter, New York, 1951.

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